Magneto Thermodynamics Stress In A Hollow Cylinder

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Abstract-

This paper is concerned an analytical method of magneto thermodynamics stress in a finite hollow cylinders under a sudden temperature change in magnetic field. The interaction between deformation and magnetic field in a hollow cylinder is considered by adding a Lorentz electro magneto force into the equation of thermo elastic motion of a hollow cylinder in an axial magnetic field. Utilizing finite integral transforms one can solve the equation of magneto thermo elastic motion and obtain the analytical expressions for the time response of magneto thermodynamics stress for a finite hollow cylinder

Keywords: *Hollow cylinders, Hankel transform, Laplace transform, magneto thermodynamics, Stresses.*

Introduction:

In this paper, an attempt has been made to determine the magneto thermo stresses in a finite hollow cylinder under a sudden temperature change in a uniform magnetic field with boundary conditions, by using the Hankel transform and Laplace transform techniques.

Nomenclature

$T(r,t)$ - Temperature charge (absolute temperature minus reference temperature).

$\tilde{U}$ - Displacement vector

U - Radial displacement

σ_r, σ_0 - Radial stress and circumferential stress.

ρ, t, a, b - Density, time and internal and external radii of a hollow cylinder coefficient of linear thermal expansion.

λ, G - Lame constants

E, v - Young's modulus and Poisson's ratio

μ - Magnetic permeability

$\tilde{H}$ - Magnetic intensity vector $_{o,o,}\ H_2$

$\tilde{e}$ - Perturbation of electric field vector.

$\tilde{h}$ - Perturbation of magnetic field vector (o, o, hz)

C_1 - Elastic wave speed

C_2 - Magnetic interference wave speed

C_L - Magneto thermo elastic wave speed

D - $(r-a)(b-a)$

τ - $(tC_L)/(b-a)$

σ_r^* - $\sigma_r/(\alpha E T(r,t))$

σ_θ^* - $\sigma_\theta/(\alpha E T(r,t))$

h_z^* - $h_z(\alpha H_2 T(r,t))$

Statement Of The Problem

Consider a long hollow cylinder with perfect conductivity placed initially in an axial magnetic field $\tilde{H}(o,o,H_z)$. Let this hollow cylinder be subjected to a rapid change in temperature $T(r,t)$ produced by the absorption of an electromagnetic pulse or γ-ray pulse radiant energy. Assuming that the magnetic permeability μ of the hollow cylinder equals the magnetic permeability of the medium around it, and omitting displacement electric current, the governing electro dynamic Maxwell equations for a perfectly conducting, elastic body are given by

$$\tilde{J} = curl\tilde{h},\quad -\frac{\mu\partial\tilde{h}}{\partial t} = curl\,\tilde{e},\quad div\tilde{h} = 0,\quad \tilde{e} = \mu\left(\frac{\partial\tilde{U}}{\partial t} x\tilde{H}\right) \quad (1)$$

Applying an initial magnetic field vector $\tilde{H}(o, o, H_z)$ in cylindrical polar co-ordinate (r, θ, z) to Eq. (1) we have

$$\tilde{U}(ucr,t),0,0), \quad \tilde{e}=\mu\left(o, H_z\frac{\partial u}{\partial t}, o\right), \quad \tilde{h}=(o,o,h_z),$$

$$\tilde{J}=\left(o, \frac{\partial h_z}{\partial r}, o\right), \quad h_z=-H_z\left(\frac{\partial r}{\partial u}+\frac{u}{r}\right)$$

From Eqs. (1), (2) and (3) the magneto elastic dynamic equation of the hollow cylinder become

$$\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}(\sigma_r-\sigma_\theta)+f_r=p\frac{\partial^2 u}{\partial t^2}$$

Where f_r is defined as

$$f_r=\mu(\tilde{J}\times\tilde{H})=\mu H_z^2\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)$$

The radial stress and the circumferential stress of a hollow cylinder subjected to a thermal shock load $T(r,t)$ are

$$\sigma_v(v,t)=(\lambda+2G)\frac{\partial u}{\partial r}+\frac{\lambda u}{r}-\frac{E\propto}{1-2v}T(r,t)$$

$$\sigma_\theta(v,t)=(\lambda+2G)\frac{u}{R}+\lambda\frac{\partial u}{\partial r}-\frac{E\propto}{1-2v}T(r,t)$$

Substituting eqs (5), (6) and (7) into eq. (4), the basic displacement equation of magneto the inelastic motion may be expressed as

$$\frac{\partial^2 u(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial u(r,t)}{\partial r}-\frac{1}{r^2}u(r,t)$$

$$=\frac{1}{C_L^2}\frac{\partial^2 u(r,t)}{\partial t^2}+\frac{E\propto}{(1-2v)\beta}\frac{\partial\tau(r,t)}{\partial r} a\le r\le b, \quad t\ge 0$$

Omitting the Maxwell tensor on the surface of the hollow cylinder, the corresponding boundary conditions are

$$\sigma_r(a,t)=\left[(\lambda+2G)\frac{\partial u}{\partial r}+\frac{\lambda u}{r}-\frac{E\propto}{1-2v}T(r,t)\right]_{r=a}=0$$

$$\sigma_r(b,t)=\left[(\lambda+2G)\frac{\partial u}{\partial r}+\frac{\lambda u}{r}-\frac{E\propto}{1-2v}T(r,t)\right]_{r=b}=0$$

The initial conditions are

$$u(r,0)=0, \quad \frac{\partial u(r,0)}{\partial t}=0$$

Solution Of The Problem

Assume that the general solution to the Eqs. (8, 9, 10, 11) may be expressed in the form

$$u(r,t)=u_t(r,t)+ud(r,t)$$

Where $u_s(r,t)$ and $u_d(r,t)$ are respectively, the (2) static and dynamic solutions to Eq. (8), (9), (10) and (11). The static solution $u_s(r,t)$ must satisfy Eq. (3) (13) and the corresponding inhomogeneous boundary conditions (9) and (10) are

$$\frac{\partial^2 us(v,t)}{\partial v^2}+\frac{1}{v}\frac{\partial us(v,t)}{\partial v}-\frac{1}{v}u^2 s(v,t)=\frac{E\propto}{(1-tv)\beta}\frac{\partial\tau(v,t)}{\partial v}$$

(4)

$$(\lambda+2G)\frac{\partial us(a,t)}{\partial v}+\frac{\lambda}{a}us(a,t)=\frac{E\propto}{(1-tv)}\tau(a,t)$$

$$(\lambda+2G)\frac{\partial us(b,t)}{\partial v}+\frac{\lambda}{b}us(b,t)=\frac{E\propto}{(1-2v)}\tau(b,t)$$ (5)

Solving Eq. (13) we have

$$u_s(v,t)=\frac{E\propto}{(1-tv)Bv}\int_a^v vT(v,t)dv+B_1 v+\frac{B_t}{v}$$

From Eqs (14) and (15) the unknown constants B_1 (6) and B_2 in Eq. (16) may be determined as

(7)

$$B_1=\frac{E\propto}{B(b^t-a^t)}\int_a^b vT(v,t)dv$$

$$B_2=\frac{E\propto a^2}{B(1-2v)(b^2-a^2)b}\int_a^b vT(v,t)dv$$

The dynamic solution, $ud(v,t)$, can be found from Eqs (8) to (15).

This solution should satisfy the (8) following inhomogeneous equation (19), the corresponding homogeneous boundary conditions (20) and (21), and the initial condition (21).

$$\frac{\partial^2 ud(v,t)}{\partial v^2}+\frac{1}{v}\frac{\partial ud(v,t)}{\partial v}-\frac{1}{v^2}ud(v,t)=\frac{1}{C_L^2}\left[\frac{\partial^2 ud(v,t)}{\partial t^2}+\frac{\partial^2 u_s(v,t)}{\partial t^2}\right]$$

(9)

$$(\lambda+2G)\frac{\partial ud(a,t)}{\partial v}+\frac{\lambda}{a}ud(a,t)=0$$

(10)

$$(\lambda+2G)\frac{\partial ud(b,t)}{\partial v}+\frac{\lambda}{b}ud(b,t)=0$$

$$ud(v,0)=-u_s(v,0)=u_0;$$

(11)

$$\frac{\partial ud(v,t)}{\partial t}=-\frac{\partial u_s(v,0)}{\partial t}=v_0$$

Where $u_s(v,t)$ is the known static solution shown in Eq. (16) the solution of the homogeneous formula of Eq. (19), assuming $u_s(v,t)=0$ is given by (12)

$$ud_0(v,t) = g(v)\exp(iwt)$$

Where $g(v)$ and w are the characteristic function and natural frequency respectively.

Substituting Eq. (23) into the homogeneous formula of Eq. (19) and utilizing Eqs (20) and (21) we have

$$\frac{d^2 g(v)}{dv^2} + \frac{1}{v}\frac{dg(v)}{dv} + (K^2 - \frac{1}{v^2})g(v) = 0, \quad a \le v \le b$$

$$(\lambda + 2G)\frac{dg(a)}{dv} + \frac{\lambda}{a}g(a) = 0$$

$$(\lambda + 2G)\frac{dg(b)}{dv} + \frac{\lambda}{b}g(b) = 0$$

The generalized solution of Eq. (2.2.24) is given by

$$g(v) = AJ_1(kv) + BY_1(kv)$$

Following Eigen equation

$$Y_a J_b - U_b J_a = 0$$

Where

$$Y_a = KnY_1^1(kna) + d_1 Y_1(kna)$$

$$J_a = KnJ_1^1(kna) + d_1 J_1(kna)$$

$$Y_b = KnY_1^1(knb) + d_2 Y_1(knb)$$

$$J_b = KnJ_1^1(knb) + d_2 J_1(knb)$$

and

$$d_1 = \frac{\lambda}{a(\lambda + 2G)}$$

$$d_2 = \frac{\lambda}{b(\lambda + 2G)}$$

Where $Jn(Knv)$ and $Yn(Knv)$ are nth-order Bessel functions of the first and second kinds, respectively. In the preceding formula, $kn(n = 1,2,\ldots,m)$ express a series of positive roots of the equation (28) and

$$wm = knc_L$$

The corresponding characteristic function (27) reduces to

$$g_n(V) = A_n Q_1(C_{nv})$$

Where

$$Q_1(k_{nv}) = J_1(k_{nv})Y_a - Y_1(k_{nv})Ja$$

By means of the normalization properly of eigen functions, the constant an Eq. (36) is determined as

$$An = \frac{\int_a^b vgn(v)Q_1(k_{nv})dv}{\int_a^b vQ_1^2(k_{nv})dv} \qquad (23)$$

Define a finite Hankel transform of $g(v)$ as

$$\bar{g}(k_n) = Hankel[g(v)] = \int_a^b (vg(v) - Q_{1(k_{nv})})dv$$

Then the inverse of Eq. (39) is given by (24)

$$g(v) = \sum_{k_n} \frac{\bar{g}(k_n)}{F(k_n)} Q_1(k_{nv}) \qquad (25)$$

Where

$$F(k_n) = \int_a^b v.Q_1^2(k_{nv})dv = \frac{t}{\pi^2 k_n^2 J_b^2}\left\{\left\{d_t^2 + k_n^2\left[1 - \left(\frac{1}{k_{nb}}\right)^2\right]\right\}J_a^2 - \left\{d_t^2 + k_n^2\left[1 - \left(\frac{1}{k_{nb}}\right)^2\right]\right\}J_b^2\right\} \qquad (26) \; (27)$$

By using Eq. (39) and performing a finite Hankel transform on Eq. (19) we have (28)

$$\frac{2J_a}{\pi J_b}\left[u_d^1(b) + d_2 ud(a)\right] - \frac{2}{\pi}\left[u_d^1(a) + d_1 ud(a)\right] - k_n^2 \bar{u}d(k_n t) \qquad (29)$$

(30)

$$= \frac{1}{C_L^2}\left[\frac{d^2\bar{u}d(k_n,t)}{dt^2} + \frac{d^2\bar{u}_s(k_n,t)}{dt^2}\right] \qquad (31)$$

(32)

Where

$$\bar{u}_s(k_n,t) = Hankel[u_s(v,t)]$$

The first and second terms on the left-hand side of Eq. (42) should be the homogeneous boundary conditions (20) and (21). Thus Eq. (42) simplifies to (33) (34)

$$-k_n^2 \bar{u}d(k_n,t) = \frac{1}{C_L^2}\left[\frac{d^2\bar{u}d(k_n,t)}{dt^2} + \frac{d^2\bar{u}_s(k_n,t)}{dt^2}\right] \qquad (43)$$

Applying Laplace transforms to Eq. (43) gives

$$\bar{u}_d^*(k_n,p) = -\bar{u}_s^*(k_n,p) + \frac{k_n^2 c_L^2}{k_n^2 c_L^2 + p^2}\bar{u}_s^* + \frac{p^2}{k_n^2 c_L^2 + p^2}\bar{u}_0 + \frac{1}{k_n^2 c_L^2 + p^2}\bar{v}_0 \qquad (44)$$

(35)

Where p is the Laplace transform parameter. Taking inverse Laplace transforms of Eq. (44), we have

$$\bar{u}_d(k_n,t) = -\bar{u}_s(k_n,t) + k_n c_L \int_0^t \bar{u}_s \sin[k_n c_L(t-\tau)]d\tau \qquad (36)$$

$$+ \bar{u}_0 \cos(k_n c_L t) + \frac{v_0}{k_n c_L}\sin(k_n c_L t) \qquad (37)$$

Using equation (40) and (41) and applying a finite inverse Hankel transform to Eq. (45), the solution $u_d(v,t)$ of Eq. (19) to (21) is expressed as

$$u_d(v,t) = \sum_{kn} \frac{\bar{u}_d(k_n,t)}{F(k_n)} Q_1(k_n v) \quad (46)$$

By substituting Eqs. (10) and (46) into Eqs. (12) the general solution of the basic equation (8) to (11) becomes

$$u(v,t) = \frac{E\alpha}{B(1-2v)v}\int_a^v vT(v,t)dv + B_1 v + \frac{B_2}{\sigma} + \sum_{k=1}^{n} \frac{\bar{u}d(k_n,t)}{F(kn)} Q_1(k_n v) \quad (47)$$

Equations (46) and (47) are the magneto thermodynamic stresses.

Conclusion

In this paper, we have investigated the magneto thermodynamic stresses in a finite hollow cylinder with the help of the finite Hankel transform and Laplace transform techniques. The expressions that are obtained can be applied to the design of useful structures or machines in engineering application.

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